Limit Definition of a Derivative

1. Consider the function defined by $f(x) = x^2 - 1$. What is the value of $\lim_{h \to 0} \dfrac{f(3+h) - f(3)}{(3+h) - 3}$?

   (a) 0

   (b) 6

   (c) 8

   (d) $2x$

   (e) The limit does not exist.

2. The expression $\lim_{h \to 0} \dfrac{(x+h)^3 - \ln(x+h) - (x^3 - \ln(x))}{h}$ is the derivative of what function?

   (a) $f(x) = (x+h)^3 - \ln(x+h)$

   (b) $f(x) = 3x^2 - \dfrac{1}{x}$

   (c) $f(x) = 3x^2 - \dfrac{1}{x}$

   (d) $f(x) = x^3 - \ln(x)$

   (e) $f(x) = \dfrac{(x+h)^3 - \ln(x+h) - (x^3 - \ln(x))}{h}$

3. What is the instantaneous rate at which the volume of an $8in^3$ cube grows as its side lengths increase from a single vertex on the left-most face?

4. If f is a differentiable function and a is a number, then $f'(a)$ is given by which of the following expressions:

   I. $\lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

   II. $\lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

   III. $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{x - h}$

   a. I only

   b. II only

   c. I and II only

   d. I and III only

   e. I, II, and III

5. The following expression represents the derivative of what function?

$$\lim_{\Delta x \to 0} \frac{2(x+\Delta x)^7 - 5(x+\Delta x) + 8 - (2x^7 - 5x + 8)}{\Delta x}$$

   a. $f(x) = 2(x+\Delta x)^7 - 5(x+\Delta x) + 8$

   b. $f(x) = 2x^7 - 5x + 8$

   c. $f(x) = 2(x+\Delta x)^7 - 5(x+\Delta x) + 8 - \left(2x^7 - 5x + 8\right)$

   d. $f(x) = 14x^6 - 5$

   e. $f(x) = \dfrac{2(x+\Delta x)^7 - 5(x+\Delta x) + 8 - (2x^7 - 5x + 8)}{\Delta x}$

6. $\lim_{h \to 0} \dfrac{(2+h)^4 - 2^4}{h} =$

   a. 0

   b. 16

   c. 1

   d. 32

   e. The limit does not exist

7. The differentiable function g is increasing over the interval $(x_0,\ x_0 + 1)$. If $x_0 < c < x_0 + 1$, what can you conclude about $\lim_{x \to c} \dfrac{g(x) - g(c)}{x - c}$? Explain your response.